

A new approach to quantum measurement problem: cluster separability

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Abstract

The paper describes a solution to the problem of quantum measurement that has been proposed recently. The literal understanding of the basic rule of quantum mechanics on identical particles violates the cluster separation principle and so leads to difficulties. A proposal due to Peres of how such difficulties could be removed is reformulated and extended. Cluster separability leads to a locality requirement on observables and to the key notion of separation status. Separation status of a microsystem is shown to change in preparation and registration processes. The indispensability of detectors plays an important role. Changes of separation status are alterations of kinematic description rather than some parts of dynamical trajectories and so more radical than 'collapse of the wave function'. Textbook quantum mechanics does not provide any information of how separation status changes run, hence new rules must be formulated. This enables to satisfy the objectification requirement for registrations. To show how the ideas work, a simplified model of registration apparatus is constructed.

1 Introduction

It is well known that the state of quantum theory of measurement is unsatisfactory. Let us mention just the excellent book [1] and more recent clearly written critical review [2].

In a series of papers [3, 4, 5, 6], we looked for solutions of three interrelated problems of quantum mechanics, those of measurement, of classical properties and of realist interpretation. The three main ideas that we proposed were very different from what had ever been published and it seems that we have been successful.

All previous attempts to formulate a realist interpretation of quantum mechanics failed because everybody was looking for objective properties where they could not be found, namely among values of observables. Quantum observables are not objective, see e.g. [1]. Their values can be obtained only after an interaction of a quantum system with a registration apparatus and so they are properties of a composite: microsystem + macrosystem, and not of the microsystem alone.

Hence, we have rejected all these attempts and proposed (an extended discussion is given in [3, 6]) instead the following:

Basic ontological hypothesis of quantum mechanics A property is objective if its value is uniquely determined by a preparation according to the rules of standard quantum mechanics. The 'value' is the value of the mathematical expression that describes the property and it may be more general than just a real number. No registration is necessary to establish such a property but a correct registration cannot disprove its value; in many cases, registrations can confirm the value.

Our interpretation of quantum mechanics differs further from Copenhagen interpretation by assuming the universality of quantum mechanics: all physical systems are quantum systems. Thus, classical systems are only a special kind of quantum systems. Our theory of classical properties is given in [4, 6]. It is based on the principle of maximum entropy rather than on the popular but incorrect ideas about coherent states. For example, we have introduced quantum states called maximum-entropy packets and shown them to match trajectories of classical mechanics better than Gaussian wave packets.

The present paper contains only a simplified version of some selected points concerning the problem of measurement.

2 Identical particles

We have to start with the standard theory of identical particles. The basic rule is:

Rule 1 *Let system \mathcal{S} consists of N particles of the same type, each particle having Hilbert space \mathbf{H} . Then, the Hilbert space of \mathcal{S} is the symmetrised tensor product of N copies of \mathbf{H} , \mathbf{H}_s^N , for bosons and the anti-symmetrised one, \mathbf{H}_a^N , for fermions. States and observables of \mathcal{S} are then described by operators on the respective Hilbert space \mathbf{H}_s^N or \mathbf{H}_a^N .*

It is known that the literal understanding of Rule 1 leads to problems (see e.g. [7] p 128). To explain the problems, let us consider two experiments.

Experiment I: State $\psi(\vec{x}_1)$ of particle \mathcal{S}_1 is prepared in our laboratory.

Experiment II: State $\psi(\vec{x}_1)$ is prepared as in Experiment I and state $\phi(\vec{x}_2)$ of particle \mathcal{S}_2 of the same type is prepared simultaneously in a remote laboratory.

Now, our laboratory claims: the state of \mathcal{S}_1 is $\psi(\vec{x}_1)$ because this has been prepared according to all rules of experimental art. An adherent of Rule 1 who knows about both laboratories will, however, claim: the true preparation is a combination of the two sub-preparations, one in each laboratory, and the state is

$$2^{-1/2}(\psi(\vec{x}_1)\phi(\vec{x}_2) \pm \phi(\vec{x}_1)\psi(\vec{x}_2)) , \quad (1)$$

as Rule 1 requires.

Thus, it seems that the notions of preparation and of state are ambiguous. In fact, Rule 1 requires to work with the state of *all* particles of the same type in the universe and the uncertainty is much higher than that in the above example.

Has this ambiguity any observable consequences? To answer this question, let us first consider Experiment I supplemented by a registration corresponding to the observable of \mathcal{S}_1 with kernel $a(\vec{x}_1; \vec{x}'_1)$, and let the registration be made in our laboratory. Measurements of this kind lead to average value

$$\int d^3x_1 d^3x'_1 a(\vec{x}_1; \vec{x}'_1) \psi^*(\vec{x}_1) \psi(\vec{x}'_1) . \quad (2)$$

Second, perform Experiment II supplemented by the same registration in our laboratory as above. The correct observable corresponding to this registration, as required by Rule 1, now is:

$$a(\vec{x}_1; \vec{x}'_1) \delta(\vec{x}_2 - \vec{x}'_2) + \delta(\vec{x}_1 - \vec{x}'_1) a(\vec{x}_2; \vec{x}'_2) . \quad (3)$$

Such measurements lead to the average value defined by Eqs. (1) and (3):

$$\int d^3x_1 d^3x'_1 a(\vec{x}_1; \vec{x}'_1) \psi^*(\vec{x}_1) \psi(\vec{x}'_1) + \int d^3x_1 d^3x'_1 a(\vec{x}_1; \vec{x}'_1) \phi^*(\vec{x}_1) \phi(\vec{x}'_1) . \quad (4)$$

Expressions (2) and (4) differ by a term that is equal to the average of observable $a(\vec{x}_1; \vec{x}'_1)$ in state ϕ . Hence, measurements on \mathcal{S}_1 are in general disturbed by other identical systems.

3 Cluster separability

Cluster separability principle is a kind of locality assumption that has been fruitful in several branches of quantum physics, see e.g. [8] and Chap. 4 of [9]. An application of the principle to identical particles can be found for instance in [7] p 128. We formulate it as follows:

Cluster Separability Principle No quantum experiment with a system in a local laboratory is affected by the mere presence of an identical system in remote parts of the universe.

Peres also observed that the principle was in general violated by results similar to those of the previous section, and he suggested a solution. This solution will now be reformulated and extended.

We introduce an important locality property of observables (for generalisation to composite systems, see [6]):

Definition 1 Let $D \subset \mathbb{R}^3$ be open. Operator with kernel $a(\vec{x}_1; \vec{x}'_1)$ is D -local if

$$\int d^3x'_1 a(\vec{x}_1; \vec{x}'_1) f(\vec{x}'_1) = \int d^3x_1 a(\vec{x}_1; \vec{x}'_1) f(\vec{x}_1) = 0 ,$$

for any test function f that vanishes in D .

Now assume for Experiment II that

1. our laboratory is inside open set $D \subset \mathbb{R}^3$,
2. $\text{supp } \phi \cap D = \emptyset$.

Then, clearly the second term in (4) vanishes for D -local observables and Eqs. (2) and (4) agree in this case (for a more general theorem see [6]). This suggests our strategy. In fact, what we shall do will formalise and explain the common practice that serenely ignores all unknown identical systems and that is intuitive, irrational but successful.

1. We introduce the key notion of our theory:

Definition 2 Let \mathcal{S} be a particle and $D \subset \mathbb{R}^3$ an open set satisfying the conditions:

- Registrations of any D -local observable \mathbf{A} of \mathcal{S} lead to average $\langle \psi(\vec{x}) | \mathbf{A} \psi(\vec{x}) \rangle$ for all states $\psi(\vec{x})$ of \mathcal{S} .
- \mathcal{S} is prepared in state $\phi(\vec{x})$ such that $\text{supp } \phi \cap D \neq \emptyset$.

In such a case, we say that \mathcal{S} has separation status D .

(For generalisation to composite systems and non-vector states, see [6].) Thus, the registration is not disturbed by other states of identical systems. This can be the case e.g. if wave functions of all other identical systems vanish in D .

2. We assume: Any preparation of \mathcal{S} must give it a non-trivial separation status $D \neq \emptyset$. Then D -local observables are individually registrable on \mathcal{S} but only these are if \mathcal{S} has not separation status D' such that D is a proper subset of D' . Indeed, registration of observables that are not D -local are then disturbed by other identical systems. Now, Peres did not warn that standard observables of quantum mechanics (position, momentum, energy, angular momentum, spin...) were not D -local with $D \neq \mathbb{R}^3$. This problem can be removed by a construction of D -local observables that are registered in real experiments and that resemble, in certain sense, the standard ones [6].
3. We list the cases in which Rule 1 does not hold:

Rule 2 *Let system \mathcal{S} be prepared in a state described by state operator T so that it has separation status $D \neq \emptyset$. Then its state is T and its observables form algebra $\mathbf{A}[\mathcal{S}]_D$ of all D -local observables of \mathcal{S} .*

Composition of such states and observables satisfy

Rule 3 *Let systems \mathcal{S}_1 and \mathcal{S}_2 be prepared in states T_1 and T_2 with separation statuses D_1 and D_2 , respectively. Then system $\mathcal{S}_1 + \mathcal{S}_2$ has state $\mathsf{T}_1 \otimes \mathsf{T}_2$ and the algebra of its observables is $\mathbf{A}[\mathcal{S}_1]_{D_1} \otimes \mathbf{A}[\mathcal{S}_2]_{D_2}$.*

Clearly, the preparation described in Rule 3 is a preparation of $\mathcal{S}_1 + \mathcal{S}_2$ in state $\mathsf{T}_1 \otimes \mathsf{T}_2$ with separation status $D_1 \cup D_2$ only if $D_1 = D_2$.

Now, cluster separability holds.

4 Preparations and registrations

As usual, we assume that any measurement on microsystems can be divided into a preparation and a registration procedure. The preparation determines a state and the registration gives a value of an observable. What do the above ideas imply for preparation and registration?

As we have seen in point 2 of our strategy, any preparation must transfer system \mathcal{S} from a trivial into a non-trivial separation status. Thus, the separation status

changes during a preparation. What is the relation of registrations to separation status change?

An important assumption of our theory of measurement is:

Rule 4 *Any registration apparatus for microsystems must contain at least one detector and every 'reading of a pointer value' (see e.g. [1]) is a signal from a detector.*

Hence, during registration, system must enter the sensitive matter of a detector. In this way, its non-trivial separation status changes into a trivial one. More discussion of this important point is given in [5, 6].

Let us give an example of separation status change. Let the states in Rule 3 be vector states of identical particles,

$$\mathbf{T}_1 = \psi(\vec{x})\psi^*(\vec{x}') , \quad \mathbf{T}_2 = \Psi(\vec{x}_1, \dots, \vec{x}_N)\Psi^*(\vec{x}'_1, \dots, \vec{x}'_N) .$$

Then the state of $\mathcal{S}_1 + \mathcal{S}_2$ is

$$\psi(\vec{x})\Psi(\vec{x}_1, \dots, \vec{x}_N) . \tag{5}$$

Let the time evolution leads to \mathcal{S}_1 entering D_2 and $\mathcal{S}_1 + \mathcal{S}_2$ having separation status D_2 . Then the state of $\mathcal{S}_1 + \mathcal{S}_2$ will be

$$\mathbf{P}_{\mathbf{S}} \circ \mathbf{P}_{s,a}^{(N+1)}(\psi(\vec{x})\Psi(\vec{x}_1, \dots, \vec{x}_N)) , \tag{6}$$

where $\mathbf{P}_{s,a}^{(N+1)}$ denotes symmetrisation or anti-symmetrisation of $N + 1$ arguments and $\mathbf{P}_{\mathbf{S}}$ is the projection to the unit sphere. The map

$$\mathbf{P}_{\mathbf{S}} \circ \mathbf{P}_{s,a}^{(N+1)} : \mathbf{H} \otimes \mathbf{H}_{s,a}^N \mapsto \mathbf{H}_{s,a}^{N+1}$$

is a non-invertible and non-linear map between two different Hilbert spaces. Second, in state (5), the observables that are registrable on \mathcal{S} form the algebra $\mathbf{A}[\mathcal{S}_1]_{D_1}$. In state (6), the algebra would be $\mathbf{A}[\mathcal{S}_1]_{\emptyset} = \emptyset$: there are no observables that would be registrable individually on \mathcal{S}_1 . Thus, the set of observables changes from $\mathbf{A}[\mathcal{S}_1]_{D_1}$ to $\mathbf{A}[\mathcal{S}_1]_{\emptyset}$.

In classical mechanics, the possible states of system \mathcal{S} are all positive normalised functions (distribution functions) on the phase space \mathbf{P} and possible observables are all real function on \mathbf{P} (at least, all such observables have definite averages on \mathcal{S} independently of external circumstances). \mathbf{P} is fixed and uniquely associated with the system alone and forms the basis of this kinematic description. Hence, transitions between different sets of observables similar to those described above would be impossible in classical mechanics. They are only enabled in quantum mechanics by the non-objective character of observables: not only their values cannot be ascribed

to microsystem \mathcal{S} alone but some of them are not even registrable in principle due to external conditions in which \mathcal{S} is.

We assume that the quantum kinematics of a microsystem is defined mathematically by the possible states represented by all positive normalised (trace one) operators, and possible observables represented by some self-adjoint operators, on the Hilbert space associated with the system. Then the transitions of states and observables that go with changes of separation status cannot be viewed as a part of a dynamical trajectory due to some new version of the dynamics of \mathcal{S} , but as a change of its kinematic description. Thus, although the change of separation status is similar to the collapse of the wave function (the non-local character included), it is both more radical and better understood.

What has been said up to now shows that textbook quantum mechanics is incomplete in the following sense:

1. It accepts and knows only two separation statuses:
 - (a) that of isolated systems, $D = \mathbb{R}^3$, with the standard operators as observables, and
 - (b) that of a member of a system of identical particles, $D = \emptyset$, with no registrable observables of its own.
2. It provides no rules for changes of separation status.

Our main idea is: Quantum mechanics must be supplemented by a theory of general separation status and by new rules that govern processes in which separation status changes. The new rules must not contradict the rest of quantum mechanics and ought to agree with and explain observational facts.

5 Extended understanding of separation status

Definition 2 leaves open the question of what the nature of disturbances is that might prevent registrations on \mathcal{S} . As yet, all examples of such a disturbance had to do with entanglement of identical particles. In the light of Rule 4, the absence of such entanglement need not be sufficient for \mathcal{S} to allow undisturbed registrations. To explain that, it is helpful to distinguish two kinds of registration.

A *direct registration* first manipulates \mathcal{S} by classical fields and shields so that the prepared beam is split into spatially separated beams, each of which associated with one eigenvalue of the registered observable. Then, there is a set of detectors each of which can be hit by only one beam. Hence, \mathcal{S} must be separated from other microsystems, not only of the same type, so that it is \mathcal{S} that is available to

those manipulation by fields and shields and has sufficient kinetic energy to excite a detector.

An *indirect registration*, such as scattering or QND measurement (see e.g. [10] and [7] p 400), lets \mathcal{S} interact with another microsystem \mathcal{S}'_1 and it is only \mathcal{S}'_1 that is then subject to a direct registration. For the measurement to be QND, several further conditions must be satisfied, but this does not concern us here. After a QND procedure, \mathcal{S} remains available to another one: another system \mathcal{S}'_2 of the same type as \mathcal{S}'_1 interacts with \mathcal{S} and is then directly registered etc. Information given by the detectors of the direct registrations reveals also something about \mathcal{S} . Thus, detectors are necessary for indirect registrations, too, and \mathcal{S} must be suitably separated from other microsystems, not only of the same type, so that we can be sure that it was \mathcal{S} that interacted with \mathcal{S}'_1 , \mathcal{S}'_2 etc.

There are interesting consequences for macroscopic systems. In general, a macroscopic system \mathcal{A} contains very many different particles. Consider the observable with kernel $a(\vec{x}_k; \vec{x}'_k)$ that concerns particle \mathcal{S} of \mathcal{A} . Suppose that there would be an apparatus \mathcal{B} associated with registration of $a(\vec{x}_k; \vec{x}'_k)$ on \mathcal{S} , if \mathcal{S} were prepared individually. Then, the apparatus \mathcal{B} cannot be applied to \mathcal{A} to register $a(\vec{x}_k; \vec{x}'_k)$ because \mathcal{S} is not suitably separated.

For example, in a direct registration, readings of \mathcal{B} are signals of a detector that registers \mathcal{S} and \mathcal{S} must be isolated to be manipulable, have sufficient kinetic energy, etc. Hence, to register $a(\vec{x}_k; \vec{x}'_k)$, we need a method that makes measurements directly on \mathcal{A} and which is, therefore, different from apparatus \mathcal{B} .

Consider a scattering as an example of indirect registration. Let \mathcal{A} be a crystal. By scattering X -rays off it, relative positions of its ions can be recognised. But rather than a position of an individual ion it is a space dependence of the average density due to all ions. In general, scattering of a microsystem \mathcal{S}' off a macrosystem \mathcal{A} can be determined in terms of potential $V_k(\vec{x}, \vec{x}_k)$ that describes the interaction between \mathcal{S}' and k -th microscopic subsystem \mathcal{S}_k of \mathcal{A} so that the whole interaction Hamiltonian is the sum

$$\sum_k V_k(\vec{x}, \vec{x}_k) .$$

Hence, the scattering yields only information on a sum of over all subsystems \mathcal{S}_k of \mathcal{A} that can interact with \mathcal{S}' and \mathcal{S}_k 's need not be identical with \mathcal{S}' . Although ingenious potentials can be invented, nature provides only a small number of potentials.

Another example is the kinetic energy of \mathcal{S} . Again, this observable cannot be measured by the method kinetic energy is measured on individual systems of type \mathcal{S} (proportional counter, scattering off a crystal, etc.). In the case that \mathcal{A} is in a state of thermodynamic equilibrium, the average kinetic energy can be calculated from the temperature. Hence, a viable method to measure the average is to measure the temperature of \mathcal{A} . Again, this is a special case that works only under specific

conditions. Further examples could be other additive quantities, such as momentum and angular momentum. Average total values of these quantities can be measured directly on \mathcal{A} . We notice that only the averages of some observables with rather large variances can be observed in these cases. It is impossible to obtain the single eigenvalues of these observables as results of registration (for an example, see Ref. [7] p 181).

Thus, the qualitative difference between macroscopic and microscopic systems is due to quantum properties of macroscopic systems rather than to some kind of limits on the validity of quantum mechanics for them.

6 Beltrametti-Cassinelli-Lahti model of quantum measurement

We shall proceed from account [1] (p 38) of model [11]. Let a discrete observable \mathbf{O} of microsystem \mathcal{S} with Hilbert space \mathbf{H} is registered. Let o_k be the eigenvalues and $\{\phi_{kj}\}$ be the complete orthonormal set of eigenvectors of \mathbf{O} ,

$$\mathbf{O}\phi_{kj} = o_k\phi_{kj} .$$

We assume that $k = 1, \dots, N$ so that there is only a finite number of different eigenvalues o_k . This is justified by the fact that no real registration apparatus can distinguish all elements of an infinite set from each other. It can therefore measure only a function of an observable that maps its spectrum onto a finite set of real numbers. Our observable \mathbf{O} is such a function.

Let the registration apparatus be a quantum macrosystem \mathcal{A} with Hilbert space $\mathbf{H}_{\mathcal{A}}$. Let \mathcal{S} and \mathcal{A} be prepared in some independent initial states and then interact for a finite time by coupling \mathbf{U} , where \mathbf{U} is a unitary transformation on $\mathcal{H}_{\mathcal{S}} \otimes \mathcal{H}_{\mathcal{A}}$. Then, a theorem has been shown in [11]:

Theorem 1 *For any initial vector state ψ of \mathcal{A} , there is a set $\{\varphi_{kl}\}$ of unit vectors in $\mathcal{H}_{\mathcal{S}}$ satisfying the orthogonality conditions*

$$\langle \varphi_{kl} | \varphi_{kj} \rangle = \delta_{lj}$$

such that \mathbf{U} is a unitary extension of the map

$$\phi_{kl} \otimes \psi \mapsto \varphi_{kl} \otimes \psi_k , \tag{7}$$

where $\{\psi_k\}$ is a set of N orthonormal vectors in $\mathbf{H}_{\mathcal{A}}$.

One assumes that states ψ_k are uniquely associated with what will be read on the apparatus after the measurement. Then, an important requirement is that the

apparatus is in one of the states $|\psi_k\rangle\langle\psi_k|$ after each individual registration. This is called *objectification requirement*.

Suppose that the initial state of \mathcal{S} is an eigenstate, $\mathsf{T} = |\phi_{kl}\rangle\langle\phi_{kl}|$, with the eigenvalue o_k . Then, Eq. (7) implies that the final state of apparatus \mathcal{A} is $|\psi_k\rangle\langle\psi_k|$, and the registration does lead to a definite result. However, suppose next that the initial state is an arbitrary vector state, $\mathsf{T} = |\phi\rangle\langle\phi|$. Decomposing ϕ into the eigenstates,

$$\phi = \sum_{kl} c_{kl} \phi_{kl} ,$$

we obtain from Eq. (7)

$$\mathsf{U}(\phi \otimes \psi) = \sum_k \sqrt{p_k} \Phi_k \otimes \psi_k , \quad (8)$$

where

$$\Phi_k = \frac{\sum_l c_{kl} \varphi_{kl}}{\sqrt{\langle \sum_l c_{kl} \varphi_{kl} | \sum_j c_{kj} \varphi_{kj} \rangle}} \quad (9)$$

and

$$p_k = \left\langle \sum_l c_{kl} \varphi_{kl} \left| \sum_j c_{kj} \varphi_{kj} \right. \right\rangle$$

is the probability that a registration of O performed on vector state ϕ gives the value o_k .

The final state of apparatus \mathcal{A} is the partial trace over \mathcal{S} :

$$\text{tr}_{\mathcal{S}}[\mathsf{U}(\mathsf{T} \otimes \mathsf{T}_{\mathcal{A}})\mathsf{U}^\dagger] = \sum_{kl} \sqrt{p_k} \sqrt{p_l} \langle \Phi_k | \Phi_l \rangle |\psi_k\rangle\langle\psi_l| . \quad (10)$$

If the objectification requirement is to be satisfied, two condition must be met:

(A) The final state of the apparatus must be the convex combination of the form

$$\text{tr}_{\mathcal{S}}[\mathsf{U}(\mathsf{T} \otimes \mathsf{T}_{\mathcal{A}})\mathsf{U}^\dagger] = \sum_j p_j |\psi_j\rangle\langle\psi_j| . \quad (11)$$

(B) The right-hand side of Eq. (11) must be the *gemenge structure* of the state.

The notion of gemenge will play an important role. The term has been introduced in Ref. [1], some authors use the term 'proper mixture' or 'direct mixture'. The crucial point is that the convex decomposition

$$\mathsf{T} = \sum_{k=1}^n w_k \mathsf{T}_k \quad (12)$$

of any state T can be a gemenge only if its preparation procedure $\mathbf{P}(\mathsf{T})$ is a random mixture with rates (frequencies) w_k of preparations $\mathbf{P}(\mathsf{T}_k)$, where each $\mathbf{P}(\mathsf{T}_k)$ is some preparation procedure for T_k , $k = 1, \dots, n$. The preparation mixture can be done by humans or result from some process in nature.

Thus, gemenge concerns a physical property of preparation rather than any mathematical characteristic of the right-hand side of Eq. (12) (such as T_k being vector states or being mutually orthogonal, etc). From the mathematical point of view, many different convex decompositions of a general state T may exist. A state is 'extremal' if it cannot be written as a non-trivial convex combination. Extremal states are described by projections onto one-dimensional subspaces of the Hilbert space. A preparation of non-extreme state T selects only some of the mathematically possible convex decompositions of T .

A random mixture of preparations is not uniquely determined by the preparation process. It can be coarsened or refined i.e. some of $\mathbf{P}(\mathsf{T}_k)$ can be combined into one preparation procedure or $\mathbf{P}(\mathsf{T}_k)$ for some k can itself be a random mixture of other preparations.

Definition 3 *The finest convex decomposition of state T defined by its preparation as gemenge is called gemenge structure of T .*

Thus, gemenge structure of T is uniquely determined by its preparation.

It may be advantageous to distinguish the mathematical convex combination of states from its gemenge structure by writing the sum in Eq. (12) as follows

$$\mathsf{T} = \left(\sum_{k=1}^n \right)_{gs} w_k \mathsf{T}_k \quad (13)$$

in the case that the right-hand side is a gemenge structure of T .

The properties that follow directly from the definition of gemenge structure and that will be needed later are described by the following theorem.

Theorem 2 1. *Gemenge structure is preserved by unitary dynamics,*

$$\mathsf{U} \left[\left(\sum_k \right)_{gs} w_k \mathsf{T}_k \right] \mathsf{U}^\dagger = \left(\sum_k \right)_{gs} w_k \mathsf{U} \mathsf{T}_k \mathsf{U}^\dagger :$$

if the sum on the left-hand side describes a gemenge structure of T , then the gemenge structure of its evolution is described by the sum on the right-hand side.

2. *In the following sense, gemenge structure is also preserved by composition of systems. Let T be a state of a composite system $\mathcal{S} + \mathcal{S}'$. The necessary and*

sufficient condition for the partial trace over \mathcal{S}' to have the gemenge structure described by

$$tr_{\mathcal{S}'}[\mathbb{T}] = \left(\sum_k \right)_{gs} w_k \mathbb{T}_k$$

is that \mathbb{T} itself has gemenge structure described by

$$\mathbb{T} = \left(\sum_k \right)_{gs} w_k \mathbb{T}_k \otimes \mathbb{T}'_k ,$$

where \mathbb{T}'_k are some states of \mathcal{S}' .

All these ideas on gemenges seem to be well known. Now, an important new point will be added: the Basic Ontological Hypothesis of Quantum Mechanics (see Introduction). It leads to a new meaning of gemenge structure: any individual system prepared in the state (13) is objectively in one of the states \mathbb{T}_k , because each of the systems has been prepared by one of the preparations $\mathbf{P}(\mathbb{T}_k)$, and the probability that $\mathbf{P}(\mathbb{T}_k)$ has been used is w_k .

It follows that the contents of both points (A) and (B) can be concisely written as

$$tr_{\mathcal{S}}[\mathbb{U}(\mathbb{T} \otimes \mathbb{T}_{\mathcal{A}})\mathbb{U}^\dagger] = \left(\sum_j \right)_{gs} p_j |\psi_j\rangle \langle \psi_j| .$$

Beltrametti-Cassinelli-Lahti model does not satisfy the objectification criterion because the end state $\mathbb{T} \otimes \mathbb{T}_{\mathcal{A}}$ of the system is $\mathbb{U}(\phi \otimes \psi)$ (Eq. (8)), which is a vector state and can therefore have only a trivial gemenge structure. Then, Point 2 of Theorem 2 implies that this is not compatible with state $tr_{\mathcal{S}}[\mathbb{U}(\mathbb{T} \otimes \mathbb{T}_{\mathcal{A}})\mathbb{U}^\dagger]$ being a non-trivial gemenge.

7 Model of registration apparatus

To show, how our ideas on measurement work, we have constructed a model of registration apparatus [5, 6]. It starts by extending and modifying Beltrametti-Cassinelli-Lahti model by additional assumptions:

1. Particle \mathcal{S} is prepared in state ϕ and separation status D .
2. \mathcal{A} is an array of N detectors \mathcal{A}_k and each detector has separation status D_k where all D_k are mutually disjoint and disjoint from D .
3. The support of state φ_{kl} is D_k for all k and l . It follows then that,

$$\langle \varphi_{kl} | \varphi_{mn} \rangle = \delta_{km} \delta_{ln} \quad k \leq N, m \leq N . \quad (14)$$

4. Each detector is composite, $\mathcal{A}_k = \mathcal{A}'_k + \mathcal{S}'_k$, where \mathcal{S}'_k consists of all particles of \mathcal{A}_k identical with \mathcal{S} . Let the number of particles in \mathcal{S}'_k be M_k and let the state of \mathcal{S}'_k be T_k . Systems \mathcal{S}'_k either are 'natural' parts of \mathcal{A}_k 's or they are created quickly after the start of the measurement by pollution.
5. \mathcal{A}'_k is the part of the sensitive matter interacting with \mathcal{S} so that ψ_k is a state of \mathcal{A}'_k .
6. State (8) is not of an end state of $\mathcal{S} + \sum_k \mathcal{A}'_k$ but a choice of an intermediate state before an amplification procedure.

The existence of systems \mathcal{S}'_k leads to a change of the separation status of \mathcal{S} after \mathcal{S} enters a detector. There are several mathematical possibilities for the choice of the intermediate state. One of the possibilities is:

Rule 5 *The intermediate state of $\mathcal{S} + \sum_k \mathcal{S}'_k + \sum_k \mathcal{A}'_k$ (before amplification) is*

$$\mathsf{T}_{int} = \left(\sum_k \right)_{gs} p_k \nu_k^2 \mathsf{T}_1 \otimes \cdots \otimes \mathsf{T}_{k-1} \otimes \mathsf{W}_{kk} \otimes \mathsf{T}_{k+1} \otimes \cdots \otimes \mathsf{T}_N \otimes |\psi_k\rangle\langle\psi_k| . \quad (15)$$

Choices:

1. The term

$$\mathsf{W}_{kk} = \mathsf{P}_{s,a}^{(M_k+1)}(|\varphi_k^1\rangle\langle\varphi_k^1| \otimes \mathsf{T}_k) \mathsf{P}_{s,a}^{(M_k+1)}$$

expresses our choice of symmetrisation or anti-symmetrisation and ν_k^2 is a normalisation factor that makes W_{kk} to a state (action of P_S).

2. Some correlations have been erased so that the state operator T_{int} is diagonal.
3. The convex decomposition (15) is postulated to be the gemenge structure of T_{int} .

The choices 2. and 3. are dictated by the objectification requirement which can be regarded as an experimental fact.

Finally, we can show that the new rule cannot be disproved by measuring the 'erased' correlations if standard quantum mechanical rules on jointly measurable observables hold [5, 6].

8 Conclusion

We have improved understanding of the theory of identical particles and removed a disorder in textbook quantum mechanics. This has lead to the notion of separation status. We have discovered the crucial role that detectors have in the theory. Then, both preparations and registrations include changes of separation status, which are changes of kinematic description and so even more radical than 'collapse of the wave function'. The theoretical freedom in changes of separation status has been used to satisfy objectification requirement. Thus, a deep revision of quantum theory of measurement results that has been derived with the help of standard principles of quantum mechanics from analysis of real measurement processes.

As yet, our results are limited to non-relativistic quantum mechanics and to measurements performed on microsystems. Moreover, only special kind of measurements have been considered, registrations of discrete observables with the help of unitary measurement couplings, the definition assumptions of Beltrametti-Cassinelli-Lahti model. Much work remains to be done.

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